Order of Operations (KCA#1)

Order of Operations
1. Perform any calculations inside parentheses or brackets.
2. Simplify exponents from left to right.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all additions and subtractions in order from left to right.

The following examples show how to simplify an expression using the order of operations.

Example 1:

$90 \div 3^2 + 7 \times 3 - 12 = 90 \div \mathbf{9} + 7 \times 3 - 12$	Exponents
$90 \div 9 + 7 \times 3 - 12 = \mathbf{10} + 7 \times 3 - 12$	Division
$10 + 7 \times 3 - 12 = 10 + \mathbf{21} - 12$	Multiplication
$10 + 21 - 12 = \mathbf{31} - 12$	Addition
$31 - 12 = \mathbf{19}$	Subtraction

Example 2:

$6 + 8[(7 - 3)^2 - (2^3 + 5)] = 6 + 8[\mathbf{4}^2 - (2^3 + 5)]$	Subtraction within parentheses
$6 + 8[\mathbf{4}^2 - (2^3 + 5)] = 6 + 8[\mathbf{4}^2 - (\mathbf{8} + 5)]$	Exponent within parentheses
$6 + 8[4^2 - (8 + 5)] = 6 + 8[4^2 - \mathbf{13}]$	Addition within parentheses
$6 + 8[4^2 - 13] = 6 + 8[\mathbf{16} - 13]$	Exponent with brackets
$6 + 8[16 - 13] = 6 + 8[\mathbf{3}]$	Subtraction with brackets
$6 + 8[3] = 6 + \mathbf{24}$	Multiplication
$6 + 24 = \mathbf{30}$	Addition

Compute with Integers (KCA #1 and #4)

Addition of Integers with the Same Sign

Add the absolute values of both integers and keep the common sign.

Example: Find -12 + (-26).

$|-12| + |-26| = 12 + 26 = 38$

Since both of the integers are negative, the result is negative. Thus,

-12 + (-26) = **-38**.

Addition of Integers with Different Signs

Subtract the smaller absolute value from the larger absolute value and give the result the sign of the integer with the larger absolute value.

Example: Find -7 + 13.

$|13| - |-7| = 13 - 7 = 6$

Since the integer with the larger absolute value (13) is positive, the final result is positive. Thus,

-7 + 13 = **6**.

SUBTRACTING WITH INTEGERS

Subtraction with integers should first be rewritten as **adding the opposite**. Then, follow the rules above for addition of integers.

Example 1: Find -18 - 13.

First, rewrite -18 - 13 as adding the opposite: **-18 + (-13)**. Then use the addition rule for adding integers with the same sign.

$|-18| + |-13| = 18 + 13 = 31$

Since both of the integers are negative, the result is negative. Thus,

-18 - 13 = -18 + (-13) = **-31**.

Example 2: Find -16 - (-42).
First, rewrite -16 - (-42) as adding the opposite: **-16 + 42**. Then use the addition rule for adding integers with different signs.

$|42| - |-16| = 42 - 16 = 26$

Since the integer with the larger absolute value (42) is positive, the final result is positive. Thus,

-16 - (-42) = -16 + 42 = **26**.

Example 3: Find 37 - (-59).

First, rewrite 37 - (-59) as adding the opposite: **37 + 59**.

Then, use the addition rule for adding integers with the same sign.

$|37| + |59| = 37 + 59 = 96$

Since both of the integers are positive, the result is positive. Thus,

37 - (-59) = 37 + 59 = **96**.

Multiplication of Positive and Negative Integers

If two integers have the same sign, then their product will be positive.

If two integers have different signs, then their product will be negative.

Example 1: Find 8 × -4.

8 × 4 = 32

Since the integers have different signs, the product is negative. Thus,
8 × -4 = **-32**.

Example 2: Find -3 × -5.

3 × 5 = 15

Since the integers have the same sign, the product is positive. Thus,

-3 × -5 = **15**.

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Division of Negative and Positive Integers

If the numerator and the denominator have the same sign, then the quotient will be positive.

If the numerator and the denominator have different signs, then the quotient will be negative.

Example 1: Find -28 ÷ 4.

28 ÷ 4 = 7

Since the numerator and the denominator have different signs, the quotient is negative. Thus,

-28 ÷ 4 = **-7**.

Example 2: Find -20 ÷ -5.

20 ÷ 5 = 4

Since the numerator and the denominator have the same sign, the quotient is positive. Thus,
-20 ÷ -5 = **4**.

Properties of Arithmetic KCA #2

The commutative, associative, distributive, inverse, and identity properties should be committed to memory.

Commutative Property:

When two real numbers are added or multiplied, the order in which the numbers are written does not affect the result.

For any real numbers *a* and *b*,

$a + b = b + a$ $\quad$ $a \times b = b \times a$

Examples:

Addition: $2 + 1 = 1 + 2$

Multiplication: $5 \times 9 = 9 \times 5$

Subtraction is not commutative: $4 - 3$ is not equal to $3 - 4$

Division is also not commutative: $6 \div 2$ is not equal to $2 \div 6$

Distributive Property:

The distributive property provides a way to change a product to a sum or a sum to a product.

For any real numbers *a*, *b*, and *c*,
$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

Examples:

$$\mathbf{3(2 + 1) = (3 \times 2) + (3 \times 1)}$$

$$\mathbf{(5 + 7)6 = (5 \times 6) + (7 \times 6)}$$

Associative Property:

An operation is associative if you can group numbers in any way without changing the answer.

For any real numbers *a*, *b* and *c*,

$$a + (b + c) = (a + b) + c$$

$$a(bc) = (ab)c$$

Examples:

Addition: (3 + 2) + 1 = 3 + (2 + 1)

Multiplication: (4 × 5) × 9 = 4 × (5 × 9)

Subtraction is not associative: (4 - 3) - 2 is not equal to 4 - (3 - 2)

Division is also not associative: (12 ÷ 2) ÷ 3 is not equal to 12 ÷ (2 ÷ 3)

Inverse Property:

For any real number *a*, there is a single real number -*a*, such that
$a + -a = 0$ and $-a + a = 0$

For any nonzero real number *a*, there is a single real number $^{1}/_{a}$ such that

$a \times {}^{1}/_{a} = 1$ and ${}^{1}/_{a} \times a = 1$

Examples:

Addition: 4 + (-4) = 0

Multiplication: $3 \times {}^{1}/_{3} = 1$

Identity Property:

For any real number *a*,
$a + 0 = a$ and $0 + a = a$

and

$a \times 1 = a$ and $1 \times a = a$

Examples:

Addition: 5 + 0 = 5

Multiplication: 9 × 1 = 9

Real Number Subsets (KCA #3)

Natural numbers are the positive counting numbers.

Natural numbers - {1, 2, 3, 4, 5, 6, 7, ...}

Whole numbers are the non-negative counting numbers.

Whole numbers - {0, 1, 2, 3, 4, 5, 6, ...}

The natural numbers are a subset of the whole numbers.

Integers are the set of numbers consisting of the whole numbers and the negative natural numbers.

Integers - {..., -3, -2, -1, 0, 1, 2, 3, ...}

The whole numbers are a subset of the integers.

Rational numbers include any number that can be written as a fraction of integers.

Rational numbers - { $^{p}/_{q}$ | p and q are integers, with $q \neq 0$}

Examples: 1, 2, 2 $^{1}/_{2}$, 3.75, 5, 6.20

The integers are a subset of the rational numbers.

Irrational numbers are the numbers that cannot be written as a fraction of integers.

Irrational numbers - {x | x is a real number that is not rational}

Examples: $\pi = 3.141592654...$, $e = 2.718281828...$, $\sqrt{2} = 1.414213562...$

Real numbers consist of the rational numbers and the irrational numbers. Natural numbers, whole numbers, integers, rational numbers, and irrational numbers are each a **subset** of the real numbers.

Real Numbers - {x | x is a rational or irrational number}

Key things to remember:

Natural Numbers $\subset$ Whole Numbers $\subset$ Integers $\subset$ Rational Numbers $\subset$ Real Numbers

The Real Numbers can be split up into Rational and Irrational Numbers.

Example problems:

1.
Which of the following groups listed below is a subset of the whole numbers?

I. Rational Numbers
II. Real Numbers
III. Natural Numbers
IV. Irrational Numbers
V. Integers

- III only
- I, III, and V only
- V only
- III and V only

2. $\left\{\frac{12}{7}, \frac{18}{7}, 6, 9, \sqrt{49}, \sqrt{121}\right\}$ (tip -remember the square root of 49 is 7, and the square root of 121 is 11. Those are not irrational numbers)

The numbers shown above belong to which of the following subsets of the real numbers?

I. Integers
II. Natural Numbers
III. Irrational Numbers
IV. Rational Numbers
V. Whole Numbers

- II, III and IV only
- All of the numbers do not belong to any one group.
- IV only
- III only

Symbolize Problem Solutions KCA # 5&9

Five Steps for Problem Solving
1. *Familiarize* yourself with the problem situation. 2. *Translate* the problem to an expression. 3. *Solve* the expression. 4. *Check* the answer in the original problem. 5. *State* the answer to the problem clearly.

This lesson concentrates on ***Familiarizing*** ourselves with the problem and ***Translating*** the word problem into an expression.

Example 1: Josh wants to buy a bicycle priced at \$127.95, but Josh only has \$86.42. How much more money does Josh need to be able to buy the bicycle?

Familiarize. Josh needs \$127.95, but he only has \$86.42. We need to find the difference.

Translate. \$127.95 - \$86.42 = Amount Needed

Example 2: During the softball season, Sarah hit 23 home runs out of 92 times at bat. What percentage of times at bat did Sarah hit a home run?

Familiarize. Sarah went to bat 92 during the softball season and hit 23 home runs. Write this as a percentage.

Translate.

$$\frac{\text{\# of home runs}}{\text{\# of times at bat}} \times 100 = \frac{23}{92} \times 100$$

Example 3: The sum of three consecutive integers 255. Find the three integers.

Familiarize. We are looking for 3 consecutive integers that add together to get 255.

Translate. Let n represent the first integer. Then n+1 is the second integer and n+2 is the third integer. Now add them together and set equal to 255.

$$n + (n+1) + (n+2) = 3n + 3 = 255$$

Real World Problems (KCA #6)

Solving word problems is one of the most difficult things to learn in math. Not only do you have to figure out what the question is asking, but you also have to apply mathematical operations correctly. Below you'll find some helpful hints to solving word problems.

UNDERSTAND THE PROBLEM

- This is the most important step.
- Identify what the problem is asking you to solve for.
- Weed out any unnecessary information given in the problem.

DEVISE A PLAN

- How will you go about solving the problem?
- Identify which skills you have learned that can be applied to solving the problem.

CARRY OUT THE PLAN

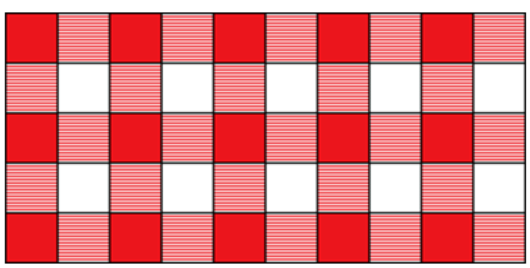
- Perform the necessary calculations to solve the problem.

REVIEW YOUR WORK

- Does your answer seem reasonable?
- Check for careless mistakes.

Examples Problems

1.



If each square of the tablecloth above measures 5 in × 5 in, what is the perimeter of the tablecloth?

150 in

1,250 in

250 in

20 in

Real World Problems

2. What is 4% of 363?

   14.52

   1,452

   145,200

   145.2

Next Question Explanation

TIP: To find the percentage of a number simply multiply the percentage times that number. Move the decimal 2 places to the left to find the convert percentage to a decimal. (4% = .04 = 4 hundredths = 4 per 100 Percent = per 100.

  363

X .04

14.52

Real World Problems

3. 349.8 is what percent of 636?

   A 0.65%

   B 65%

   C 55%

   D 0.55%

? x 636 = 349.8 $\rightarrow$ 349.8 / 636 = .55 (55%)

Linear Equations (KCA #7)

Operations can be performed on equations to simplify as long as the operations are performed on both sides of the equation. This keeps the equation **balanced**.

EXAMPLES

1. In the equation below, 8 is subtracted from both sides in order to simplify the equation.

Equation:	$\mathbf{y} + 8$	$=$	10
8 is subtracted from both sides:	$\mathbf{y} + 8 - 8$	$=$	$10 - 8$
The equation is simplified:	$\mathbf{y}$	$=$	2

2. In the equation below, 3 is added to both sides in order to simplify the equation.

Equation:	$\mathbf{y} - 3$	$=$	7
3 is added to both sides:	$\mathbf{y} - 3 + 3$	$=$	$7 + 3$
The equation is simplified:	$\mathbf{y}$	$=$	10

3. In the equation below, both sides of the equation are multiplied by 4 in order to simplify the equation.

Equation:	$^{\mathbf{y}}/_{4}$	$=$	$3 + 2$
Both sides multiplied by 4:	$^{\mathbf{y}}/_{4} \times 4$	$=$	$(3 + 2) \times 4$
The equation is simplified:	$\mathbf{y}$	$=$	20

Remember to check the answers by substituting the value of the variable back into the original equation.

Linear Equations (KCA # 8 & 11)

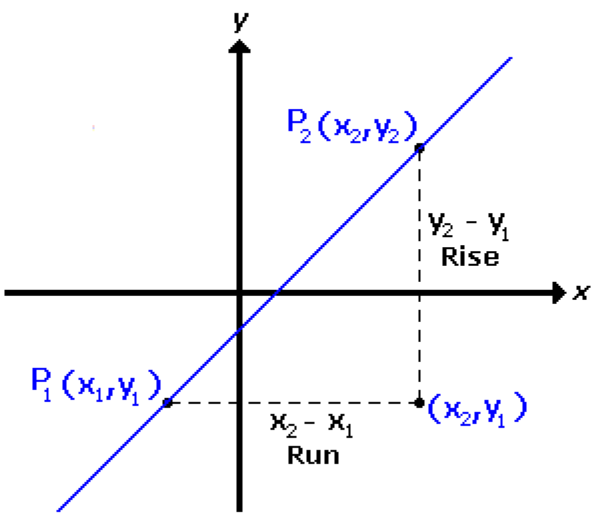
The **graph of a linear equation** is the line made up of the set of points whose coordinates satisfy the equation in two variables.

The **slope** of a line is a measure of how steep the line is.

Given a line in the plane, the ratio of the change in y to the change in x as you move from left to right is the slope of the line.

If a line passes through two distinct points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ where $x_1 \neq x_2$, then its **slope** is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}}$$



Example: Find the slope of the line that passes through the points (-4, 3) and (2, 1).

$$\textit{slope } m = \frac{1 - 3}{2 - (-4)} = \frac{-2}{6} = \frac{-1}{3}$$

A line that goes up from left to right has a **positive** slope.
A line that goes down from left to right has a **negative** slope.
A horizontal line has a slope of zero.
A vertical line has an undefined slope.

The equation **y = mx + b** is called the **slope-intercept form** of a linear equation, where **m** is the slope of the line and **b** is the y-intercept.

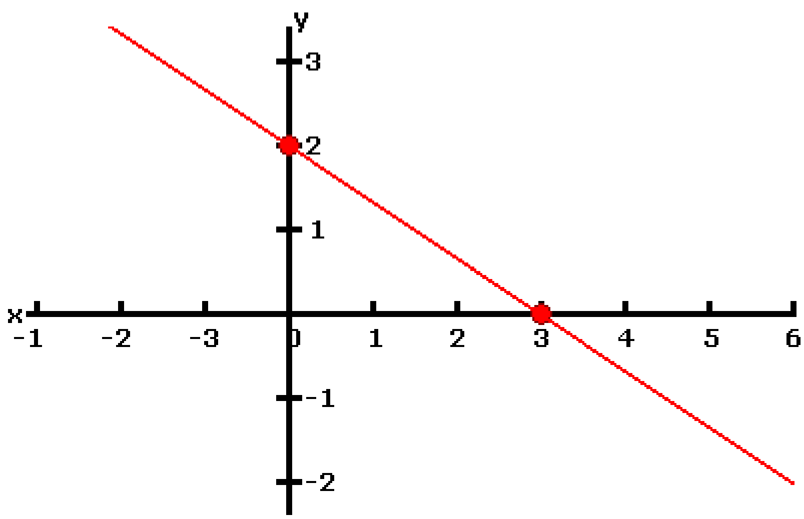
Example 1:

Graph equation: $\mathbf{y = (^{-2}/_{3})x + 2}$

The y-intercept of the equation is 2, so first put a point on the graph at (0,2).

Then, the slope of the equation is $^{-2}/_{3}$, so from the point (0,2) put the next point 2 down and 3 to the right at (3,0).

Connect the two points with a straight line.



The **x-intercept** for a linear equation can easily be found by substituting zero into the equation for y.

Example 2:

What is the **x-intercept** for the linear equation: **y = 3x + 9**

Substitute zero in for **y**: 0 = 3x + 9

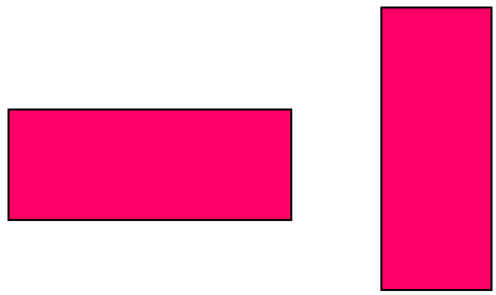
Solve the equation for **x**: -3 (x) = 9 **x** = -3

Similar & Congruent Figures KCA #10

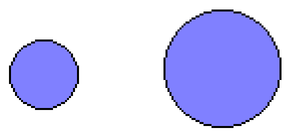
If you can flip, rotate, or move one shape to fit exactly on another, the shapes are **congruent**. Two figures are **congruent** if they are the same size and shape.

Examples:

The two shapes to the right are **congruent**:

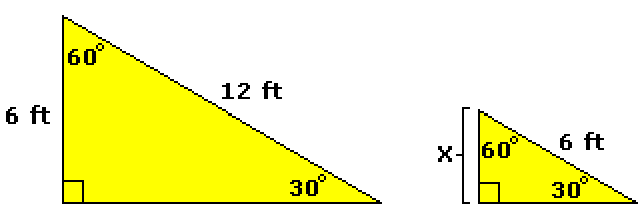


The two shapes to the right are **NOT congruent**:



Two shapes are **similar** if their corresponding angles are equal and their corresponding line segments are proportional.

Example:

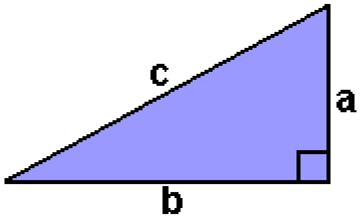


Question: The two triangles above are similar because their corresponding angles are congruent. What is the length of side x on the smaller triangle?

Answer: The sides of similar figures are always proportional, so

12/6 = 6/x
x = 3 ft

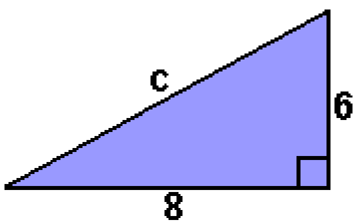
Pythagorean Theorem (KCA # 12)



Pythagorean Theorem
$a^2 + b^2 = c^2$

The **Pythagorean theorem** applies to right triangles (and only right triangles), where **c** is the length of the hypotenuse (the side opposite the right angle) and **a** and **b** are the lengths of the legs (the other two sides).

Example 1:



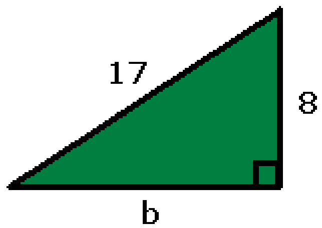
In the triangle above, what is the length of side c?

Solution:

Use the Pythagorean theorem.

$$\begin{aligned} 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2 \\ \sqrt{100} &= c \\ 10 &= c \end{aligned}$$

Example 2:



In the triangle above, what is the length of side b?

Solution:

Use the Pythagorean theorem.

$$8^2 + b^2 = 17^2$$

$$64 + b^2 = 289$$

$$b^2 = 225$$

$$b^2 = \sqrt{225}$$

$$b = 15$$

Mean, Median, and Mode (KCA # 13)

The **mean**, **median**, and **mode** are all used to calculate the "middle" of a set of data.

Calculating Mean: Add up the numbers then divide by the number in the set to get the mean.

Example: Find the mean of the following: { 66, 72, 83, 89 }

$$\textbf{Mean} = \frac{66 + 72 + 83 + 89}{4}$$

$$\textbf{Mean} = \frac{310}{4}$$

$$\textbf{Mean} = 77.5$$

Calculating Median: Put the numbers in order from smallest to largest. The middle number is the median.

Example: Find the median of the following: { 65, 72, 81, 83, 89 }

Median = the middle number from smallest to largest

Median = 81

Calculating Mode: The mode is the number that appears most often in the set.

Example: Find the mode of the following: { 65, 65, 71, 72, 81, 83, 83, 83, 89 }

Mode = the number that appears most often

Mode = 83

Probability (KCA #14)

Probability refers to the chance that an event will happen.

Probability is presented as the ratio of the number of ways an event can occur relative to the number of possible outcomes.

$$\text{Probability of Event} = \frac{\text{Number of ways an event can occur}}{\text{Number of possible outcomes}}$$

Example 1: If you roll a die, what's the probability of rolling a four?

$$P(4) = \frac{\text{Number of ways an event can occur: } \{4\}}{\text{Number of possible outcomes: } \{1,2,3,4,5,\text{or } 6\} \text{ outcomes}}$$

P(4) = $\mathbf{^{1}/_{6}}$

Example 2: If you roll a die, what's the probability of rolling a number less than 4?

$$P(\text{Less Than } 4) = \frac{\text{Number of ways an event can occur: } \{1,2,\text{or } 3\}}{\text{Number of possible outcomes: } \{1,2,3,4,5,\text{or } 6\}}$$

P(Less than 4) = $^{3}/_{6}$ = $\mathbf{^{1}/_{2}}$

Example 3: If you pick from a bag that contains 5 blue marbles, 2 green marbles, and 3 red marbles, what's the probability of picking a red marble?

$$P(\text{red marble}) = \frac{\text{Number of ways an event can occur: } \{3 \text{ red}\}}{\text{Number of possible outcomes: } \{5 \text{ blue} + 2 \text{ green} + 3 \text{ red}\}}$$

P(red marble) = $\mathbf{^{3}/_{10}}$

Probability of A and B

If event A and event B are independent, the probability of both event A and event B occurring is P(A) × P(B).

Example 1: If you flip a coin twice, what's the probability that heads comes up both times?

P(heads 1st flip) = $^{1}/_{2}$
P(heads 2nd flip) = $^{1}/_{2}$

Since P(heads 1st flip) and P(heads 2nd flip) are independent events:

P(heads both flips) = $^{1}/_{2} \times {}^{1}/_{2} = \mathbf{^{1}/_{4}}$

Example 2: If you roll 2 dice, what's the probability that 4 comes up on both dice?

P(4 on 1st roll) = $^{1}/_{6}$
P(4 on 2nd roll) = $^{1}/_{6}$

Since P(4 on 1st roll) and P(4 on 2nd roll) are independent events:

P(4 on both rolls) = $^{1}/_{6} \times {}^{1}/_{6} = \mathbf{^{1}/_{36}}$