KCA #1 Compute Decimals

KCA #1 Fractions & Decimals 7.1.4.k2

Adding Decimals

- Place the numbers so that the **decimal points are aligned vertically**.
- Add each column, starting on the right and working left. If the sum of a column is greater than ten, then carry the one to the next column on the left.

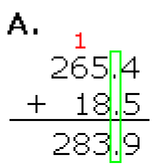
Examples:

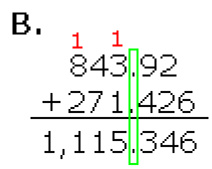
A. 265.4 + 18.5 =

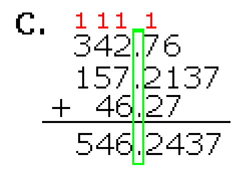
B. 843.92 + 271.426 =

C. 342.76 + 157.2137 + 46.27 =

Solution







Subtracting Decimals

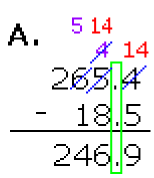
- Place the numbers so that the **decimal points are aligned vertically**.
- Subtract each column, starting on the right and working left. If the number being subtracted is larger than the number it is being subtracted from, then add ten to the number and subtract one from the number in the next left column. This is called **borrowing**.

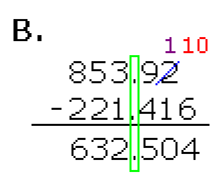
Examples:

A. 265.4 - 18.5 =

B. 853.92 - 221.416 =

Solution





Multiply and Divide Decimals

The most important thing to remember when multiplying or dividing decimal numbers is knowing the decimal placement in the solutions.

Multiplying Decimal Numbers

When multiplying decimal numbers, multiply the numbers together as if there was not a decimal point in the problem. Then, move the decimal point ***to the left*** in the solution the same number of places that are ***to the right*** of the decimal points in the problem.

Example: 3.45 × 3 = ?

First, rewrite the problem vertically. Then take out the decimals.

Step 1: Rewrite without decimals.

$$\begin{array}{r} 3.45 \\ \times \quad 3 \\ \hline \end{array} = \begin{array}{r} 345 \\ \times \quad 3 \\ \hline \end{array}$$

Step 2: Solve.

$$\begin{array}{r} 345 \\ \times \quad 3 \\ \hline 1035 \end{array}$$

Step 3: Move the decimal point two places to the left in the solution because there are 2 numbers to the right of the decimal point in the problem (in 3.45, both 4 and 5 are to the right of the decimal point).

$$\begin{array}{r} 345 \\ \times \quad 3 \\ \hline 1035. \end{array}$$

Solution: 10.35

Dividing Decimal Numbers

Example: 2.56 ÷ 4 = ?

First, rewrite the problem as a long division problem. Then move the decimal straight up. Finally, divide.

Step 1: $4\overline{)\,2.56}$

Step 2:
$$\begin{array}{r} .64 \\ 4\overline{)\,2.56} \\ -2\,4 \\ \hline 16 \\ -16 \\ \hline 0 \end{array}$$

Solution: 0.64

KCA #2 Algebraic Expressions

KCA #2 Evaluates Simple Algebraic Expressions 7.2.2.k8

When evaluating an algebraic expression for given variable values, substitute into the expression and then follow the correct order of operations.

1. Parentheses or Brackets from the inside out.
2. Exponents
3. Multiplication and Division from left to right.
4. Addition and Subtraction from left to right.

Example 1. If ***a* = 5** and ***b* = 2**, what is the value of **10(*a*+3*b*) - 3**?

10(*a* + 3*b*) - 3	=	10(5 + 3(2)) - 3	Substitute values in for *a* and *b*.
	=	10(5 + 6) - 3	Multiplication in parentheses.
	=	10(11) - 3	Addition in parentheses.
	=	110 - 3	Multiplication.
	=	**107**	Subtraction.

KCA #3 Symbolize Problem Solutions

KCA #3, Write linear expressions 7.2.2.a1

Five Steps for Problem Solving

1. *Familiarize* yourself with the problem situation.
2. *Translate* the problem to an expression.
3. *Solve* the expression.
4. *Check* the answer in the original problem.
5. *State* the answer to the problem clearly.

This lesson concentrates on **familiarizing** ourselves with the problem and **translating** the word problem into an expression.

Example 1:
Seth is 12 years old. Jessica is 9 more than twice Seth's age. How old is Jessica?

Familiarize. We want to find Jessica's age given that she is 9 more than twice Seth's age.

9 more than twice Seth's age = 2 times Seth's age plus 9.

Translate. Let S represent Seth's age. Then

2 times Seth's age plus 9 = $2 \times S + 9 = 2 \times 12 + 9$.

Example 2:
Josh wants to buy a bicycle priced at $127.95, but Josh only has $86.42. How much more money does Josh need to be able to buy the bicycle?

Familiarize. Josh needs $127.95, but he only has $86.42. We need to find the difference.

Translate. $127.95 - $86.42 = Amount Needed

Example 3:
Bernice swam 1,750 m yesterday at the school's swimming pool. One lap at the pool is 350 m. Bernice wants to swim 2,800 m today. How many laps around the pool should Bernice swim today?

Familiarize. Bernice wants to swim 2,800 m and each lap is 350 m. We need to find how many laps equal 2800 m.

Translate. Let L equal one lap around the pool. Then we want to solve

$$350L = 2{,}800$$

KCA #4 Notation - Equivalent Representations

KCA #4, Solves Real-World Problems 7.1.1.a1
A number is written in **scientific notation** when it is expressed in the form $a \times 10^n$ where a is greater than or equal to 1 and less than 10 and n is an integer.

Example 1: Write 73,200 in scientific notation.

To write 73,200 as a number greater than or equal to 1 and less than 10, move the decimal point 4 places to the left. To offset moving the decimal point, multiply by 10^4.

Thus, 73,200 = $\mathbf{7.32 \times 10^4}$.

Example 2: Write 0.0000732 in scientific notation.

To write 0.0000732 as a number greater than or equal to 1 and less than 10, move the decimal point 5 places to the right. To offset moving the decimal point, multiply by 10^{-5}.

Thus, 0.0000732 = $\mathbf{7.32 \times 10^{-5}}$.

To add or subtract numbers in scientific notation, first convert one of the numbers so that both numbers have the same power of ten. Next, add or subtract the numbers multiplied by powers of 10 and keep the power of 10 the same. If necessary, move the decimal point and change the exponent of 10 so that the answer is in scientific notation.

Example 3: Simplify the expression, $1.31 \times 10^5 + 3.27 \times 10^3$.

First, write 1.31×10^5 in terms of 10^3.

$$1.31 \times 10^5 = 131 \times 10^3$$

Next, add the numbers by adding the numbers multiplied by the power of ten and keeping the power of 10 the same.

$$131 \times 10^3 + 3.27 \times 10^3 = (131 + 3.27) \times 10^3$$
$$= 134.27 \times 10^3$$

Write the answer in scientific notation.

$$134.27 \times 10^3 = \mathbf{1.3427 \times 10^5}$$

Example 4: Simplify the expression, $4.26 \times 10^{-2} - 5.96 \times 10^{-3}$.

First, write 4.26×10^{-2} in terms of 10^{-3}.

$$4.26 \times 10^{-2} = 42.6 \times 10^{-3}$$

Next, subtract the numbers by subtracting the numbers multiplied by the power of ten and keeping the power of 10 the same.

$$42.6 \times 10^{-3} - 5.96 \times 10^{-3} = (42.6 - 5.96) \times 10^{-3}$$
$$= 36.64 \times 10^{-3}$$

Write the answer in scientific notation.

$$36.64 \times 10^{-3} = \mathbf{3.664 \times 10^{-2}}$$

To multiply numbers in scientific notation, multiply the numbers multiplied by the power of ten and add the exponents of 10. To divide numbers in scientific notation, divide the numbers multiplied by the power of ten and subtract the exponents of 10. If necessary, move the decimal point and change the exponent of 10 so that the answer is in scientific notation.

Example 5: Simplify the expression, $(5.8 \times 10^{-4}) \times (1.7 \times 10^{-4})$.

First, group the numbers multiplied by the power of ten and the powers of 10.

$$(5.8 \times 10^{-4}) \times (1.7 \times 10^{-4}) = (5.8 \times 1.7) \times (10^{-4} \times 10^{-4})$$

Multiply the numbers in the first parentheses and add the exponents of 10 in the second parentheses.

$$(5.8 \times 1.7) \times (10^{-4} \times 10^{-4}) = 9.86 \times 10^{(-4 + (-4))}$$
$$= \mathbf{9.86 \times 10^{-8}}$$

Example 6: Simplify the expression, $(5.8 \times 10^{-4}) \div (1.6 \times 10^{-4})$.

First, group the numbers multiplied by the power of ten and the powers of 10.

$$(5.8 \times 10^{-4}) \div (1.6 \times 10^{-4}) = (5.8 \div 1.6) \times (10^{-4} \div 10^{-4})$$

Divide the numbers in the first parentheses and subtract the exponents of 10 in the second parentheses.

$(5.8 \div 1.6) \times (10^{-4} \div 10^{-4})$ $= 3.625 \times 10^{(-4 - (-4))}$

$= 3.625 \times 10^{0} = 3.625 \times 1$

$= 3.625$

Expanded Notation

Numbers can be written in many different formats. You need to know how to convert between the different formats.

Standard Notation and Expanded Notation

Standard Notation

Standard notation is the most common format for writing numbers. The following numbers are all in standard notation.

12
2,176
345
3,423,001.03

Expanded Notation

A number written in expanded notation is broken down into parts just like it is in a place-value table.

5,204.49 written in expanded notation is:

5,000 + 200 + 4 + 0.4 + 0.09

KCA #5 Percentage

KCA #5, Percentages 7.1.4.k5

Example: The price of a video game is $50. If the sales tax is 10%, what is the total cost of one video game?

Answer: Sales tax is 10% of $50. To figure out the tax on $50, multiply $50 by 10%. $50 × 0.10 = $5

Now add the tax onto the original cost.

$50 + $5 = **$55**

Converting Fractions, Decimals, & Percents

Converting from a **decimal** to a **percent**: MULTIPLY the decimal **by 100**.

Converting from a **percent** to a **decimal**: DIVIDE the percent **by 100**.

Examples:

Convert **0.25** to a **percent:** (0.25 × 100) = 25%

Convert **0.75** to a **percent:** (0.75 × 100) = 75%

Convert **25%** to a **decimal:** (25 ÷ 100) = 0.25

Convert **75%** to a **decimal:** (75 ÷ 100) = 0.75

Fractions, decimals, and percents can all be used to represent part of a whole. It is important to know how to convert common fractions and common percents. See the table of common fractions and percents below.

Fraction = Decimal = Percent
$^{1}/_{10}$ = 0.1 = 10%
$^{1}/_{5}$ = 0.2 = 20%
$^{1}/_{2}$ = 0.5 = 50%
$^{3}/_{4}$ = 0.75 = 75%
$^{4}/_{5}$ = 0.8 = 80%
$^{9}/_{10}$ = 0.9 = 90%

Real World Problems

Solving word problems is one of the most difficult things to learn in math. Not only do you have to figure out what the question is asking, but you also have to apply mathematical operations correctly. Below you'll find some helpful hints to solving word problems.

UNDERSTAND THE PROBLEM

- This is the most important step.
- Identify what the problem is asking you to solve for.
- Weed out any unnecessary information given in the problem.

DEVISE A PLAN

- How will you go about solving the problem?
- Identify which skills you have learned that can be applied to solving the problem.

CARRY OUT THE PLAN

- Perform the necessary calculations to solve the problem.

REVIEW YOUR WORK

- Does your answer seem reasonable?
- Check for careless mistakes.

Real World Problems

1. Anita is assembling a standing lamp for her living room. The lamp is made of three long pieces that will connect end-to-end.

The base section is 0.5 meter long, the middle section is 2.1 meters long, and the top section is 1.3 meters long. How tall will the lamp be when it is assembled?

3.8 meters

5.1 meters

3.9 meters

2.7 meters

Explanation: In order to find the total height of the lamp after it is assembled, add up the lengths of the three sections.

$$0.5 \text{ meter} + 2.1 \text{ meters} + 1.3 \text{ meters} = 3.9 \text{ meters}$$

So, after it is assembled, the lamp will be a total of **3.9 meters** tall.

KCA #6 KCA #6, Relates Ratios, Proportions & Percents 7.2.2.k7

Ratios, Proportions, & Percents

A **ratio** represents a comparison between two values.

A ratio of two numbers can be expressed in three ways. A ratio of "one to two" can be written as:

1. 1 to 2
2. 1:2
3. $^1/_2$

Example: There are 7 boys and 8 girls in Mrs. Hamilton's math class. What is the ratio of girls to students?

Solution: There are 8 girls and 7 + 8 = 15 total students. So, the ratio of girls to students is **8 to 15** or **8:15**.

A **proportion** is made up of two ratios with an "=" (equal) sign between them. For example:

$$\frac{1}{2} = \frac{4}{8}$$

Example: Billy is building a model of a 12 foot long truck. The model is $^3/_8$ the size of the truck. How long is Billy's model?

Solution: First make a proportion for the situation with one unknown:

$$\frac{\mathbf{x}}{12} = \frac{3}{8}$$

To solve for an unknown in a proportion:

1. Cross-multiply: multiply the numbers that are diagonal to each other.
2. Set the two products equal to each other.
3. Divide both sides by the number next to the x.

$$\frac{\mathbf{x}}{12} = \frac{3}{8}$$

$8(\mathbf{x})$	=	12(3)	Cross multiply
$8\mathbf{x}$	=	36	
$\mathbf{x}$	=	$^{36}/_8$	Divide by 8
$\mathbf{x}$	=	**4.5**	

Therefore, Billy's model is 4.5 feet long.

A **percent** is a ratio whose second term is 100. Percent means parts per hundred. In mathematics, we use the symbol % for percent. Percents can easily be converted to a decimal by dividing by 100. Example: 20% = 0.2

Example: The price of a hamburger \$2.50. If the sales tax is 8%, what is the total cost of one hamburger?

Solution: To figure out the tax on \$2.50, multiply \$2.50 by 8%.

$$\$2.50 \times 0.08 = \$0.20$$

Now add the tax onto the original cost.

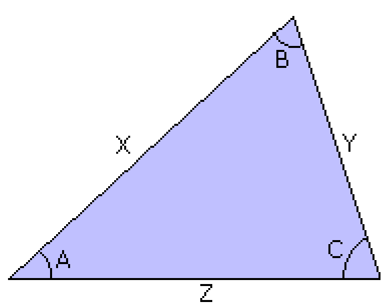
$$\$2.50 + \$0.20 = \mathbf{\$2.70}$$

KCA #7

Properties of Triangles & Quadrilaterals 7.3.1.k3

Angles in Triangles

The sum of the measure of the angles of a triangle always equals **180°**.



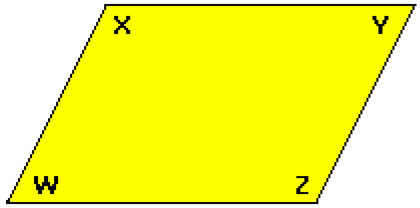
If **A** equals 50° and **B** equals 55°, what is the value of **C**?

Use the fact that the sum of all angles in a triangle is 180°.

$$\begin{aligned} 50° + 55° + \mathbf{C} &= 180° \\ \mathbf{C} &= 180° - 55° - 50° \\ \mathbf{C} &= 75° \end{aligned}$$

Angles in Quadrilaterals

The sum of the measure of the angles of a quadrilateral always equals **360°**.



If **X** and **Z** equal 110° each, what is the value of angle **W** and **Y**?

You know that the sum of all angles is 360°, and that **W** and **Y** are equal since it is a parallelogram:

$$110° + 110° + 2C = 360°$$
$$2C = 360° - 220°$$
$$2C = 140°$$
$$C = 70°$$

So, angle **W** and angle **Y** each measure **70°**.

Quadrilaterals

A **quadrilateral** is a polygon with four sides and four vertices. The sum of the interior angles of a quadrilateral is 360°.

Classifications of Quadrilaterals

Trapezoid - one pair of opposite sides that are parallel.

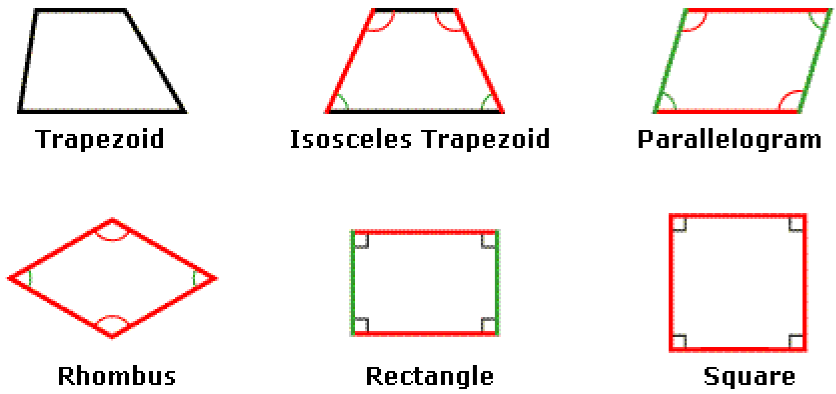
Isosceles Trapezoid - two of the opposite sides are parallel, the two other sides are equal, and the two ends of each parallel side have equal angles.

Parallelogram - both pairs of opposite sides are parallel.

Rhombus - four sides of equal length.

Rectangle - each angle is a right angle.

Square - four sides have equal length, and each angle is a right angle.



Notice that:

A square is also a rectangle, rhombus, and parallelogram.

A rectangle is a parallelogram.

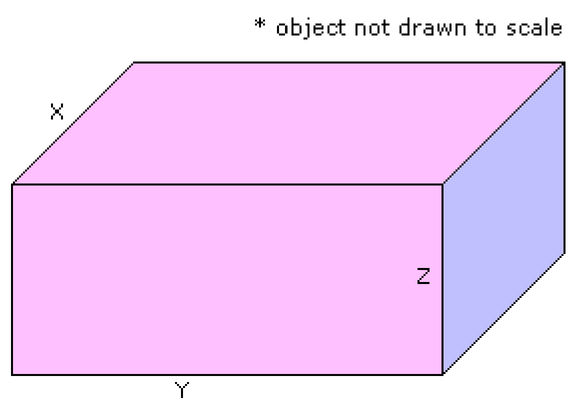
A rhombus is a parallelogram.

KCA #9 Surface Area & Volume

KCA #9 Surface Area & Volume 7.3.2.k6
Surface area is the 2-dimensional area that is on the outside of an object. In other words, it is the area of the surface of an object. To calculate the surface areas of objects with flat faces (prisms, pyramids, etc), add up the area of each face of the object.

Example:

Question:



If X=2 cm, Y=3 cm, and Z=4 cm, what is the **surface area** of the box above?

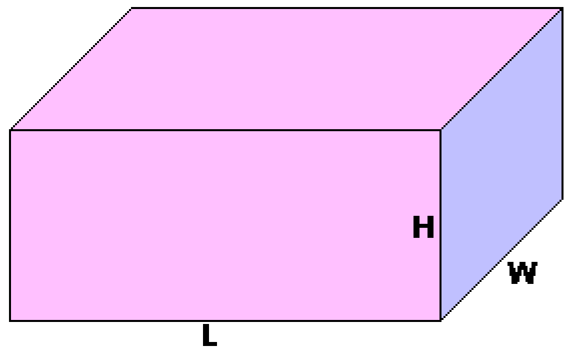
Answer: Find the area of each face, and then add them all together.

Surface area = $\mathbf{(3 \times 2) + (3 \times 2) + (3 \times 4) + (3 \times 4) + (2 \times 4) + (2 \times 4)}$

Surface area = $\mathbf{6 + 6 + 12 + 12 + 8 + 8}$

Surface area = $\mathbf{52\ cm^2}$

The **volume** of a solid is the number of cubic units contained in the solid.



The box above has the width (W), length (L) and height (H). The **volume** of the box is equal to the product of the area of the base (L x W) and the height (H).

Volume = L × W × H

Example: Find the **volume** of a box with length 5, width 4 and height 6.

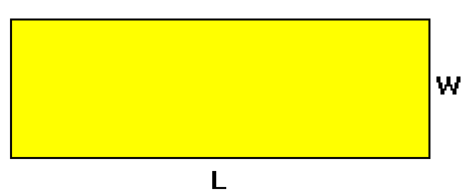
Volume = L × W × H = 4 × 5 × 6

Area = 120 cubic units

KCA #8 & KCA #11, Perimeter & Area Formulas & for Composite Figures 7.3.2.k4 and 7.3.2.a1

KCA #8 and #11 Perimeter

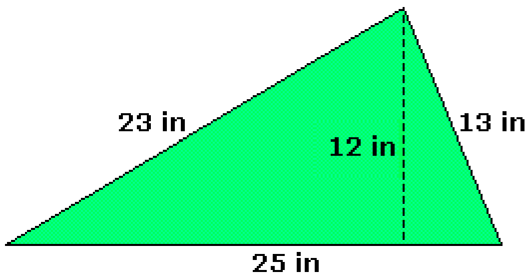
Perimeter is the distance around a flat (2-dimensional) shape.



Example:

If the rectangle shown above has a length of 15 cm and a width of 5 cm, what is the perimeter?

Solution:
Perimeter = 15 cm + 5 cm + 15 cm + 5 cm
= **40 cm**



Example:

What is the perimeter of the triangle pictured above?

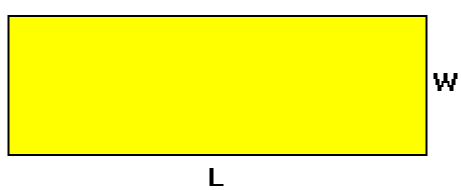
Solution:
Perimeter = 25 in + 13 in + 23 in
= **61 in**

Area

Area is the amount of space taken up by a flat (2-dimensional) shape.

There are two area formulas that are used often and need to be committed to memory. The formulas for the area of a rectangle and the area of a triangle.

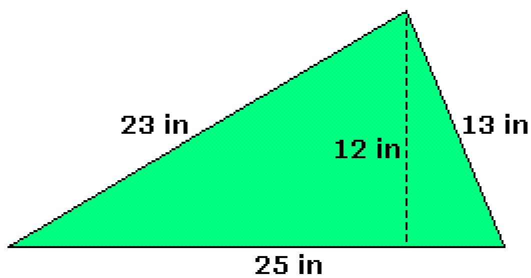
Area of a Rectangle = length × width



Example: The rectangle shown above has a length of 12 cm and a width of 4 cm. Find the **area** of the rectangle.

Area = length × width
= 12 cm × 4 cm
= **48 cm^2**

Area of a Triangle = $^1/_2$ × base × height



Example: Find the **area** of the triangle above.

Area **½** X 25 in X 12 in
12.5 X 12 in = 150 in

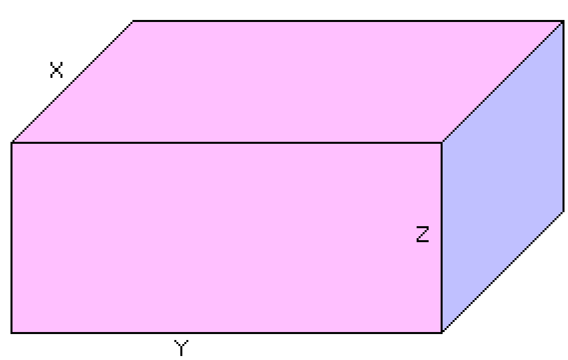
KCA #9 Surface Area & Volume

Surface area is the 2-dimensional area that is on the outside of an object. In other words, it is the area of the surface of an object.

To calculate the surface areas of objects with flat faces (prisms, pyramids, etc), add up the area of each face of the object.

Example:

Question:



If X=2 cm, Y=3 cm, and Z=4 cm, what is the **surface area** of the box above?

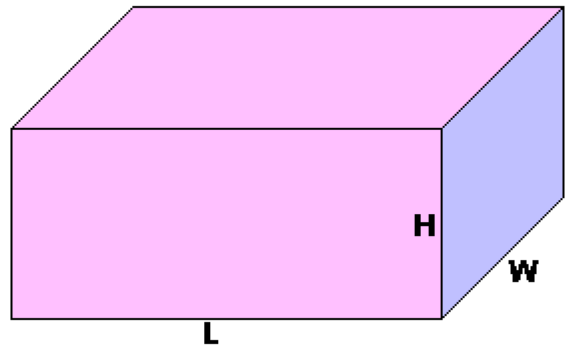
Answer: Find the area of each face, and then add them all together.

Surface area = **(3 × 2) + (3 × 2) + (3 × 4) + (3 × 4) + (2 × 4) + (2 × 4)**

Surface area = **6 + 6 + 12 + 12 + 8 + 8**

Surface area = **52 cm^2**

The **volume** of a solid is the number of cubic units contained in the solid.



The box above has the width (W), length (L) and height (H). The **volume** of the box is equal to the product of the area of the base (L x W) and the height (H).

Volume = L × W × H

Example: Find the **volume** of a box with length 5, width 4 and height 6.

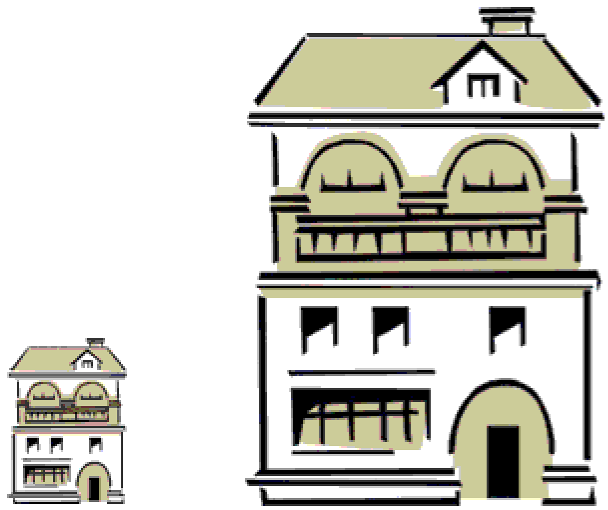
Volume = L × W × H = 4 × 5 × 6

Area = 120 cubic units

KCA #10 Scale Drawings

Large objects can be represented by smaller **scale drawings**. The proportions of the objects, or scale, must be consistent throughout the drawing.

Example 1:



1 cm = 15 feet   1 cm = 5 feet

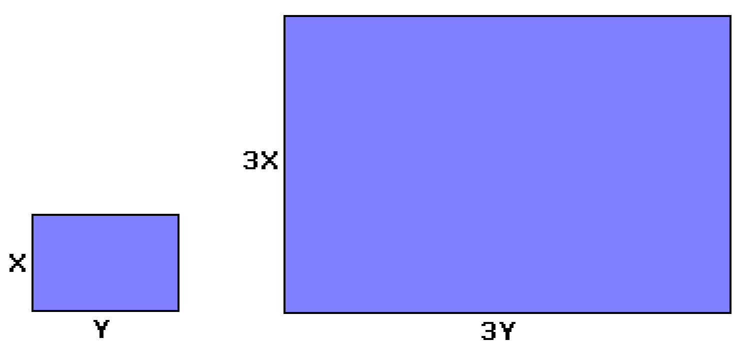
The two drawings above are scale drawings of the same house. If the smaller drawing on the left measures 2 cm tall, how many cm does the larger house on the right measure?

Answer: In the smaller drawing, each centimeter stands for 15 feet, which is 3 times 5 feet. So, it takes 3 times as many centimeters to equal a foot in the smaller drawing than it takes to equal a foot in the larger drawing. This means that the larger drawing is 3 times larger than the smaller drawing. Multiply the size of the smaller house (2 cm) by 3.

$$2 \text{ cm} \times 3 = 6 \text{ cm}$$

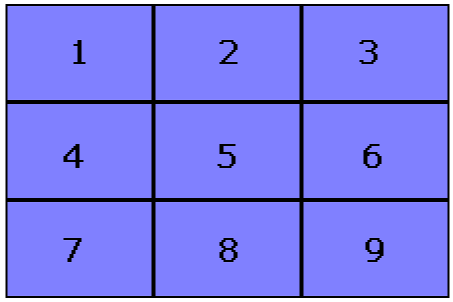
So, the larger house measures **6 cm**.

Example 2:



If the **area** of the smaller rectangle is 10 $feet^2$, what is the **area** of the larger rectangle?

Answer: Look at the image below:



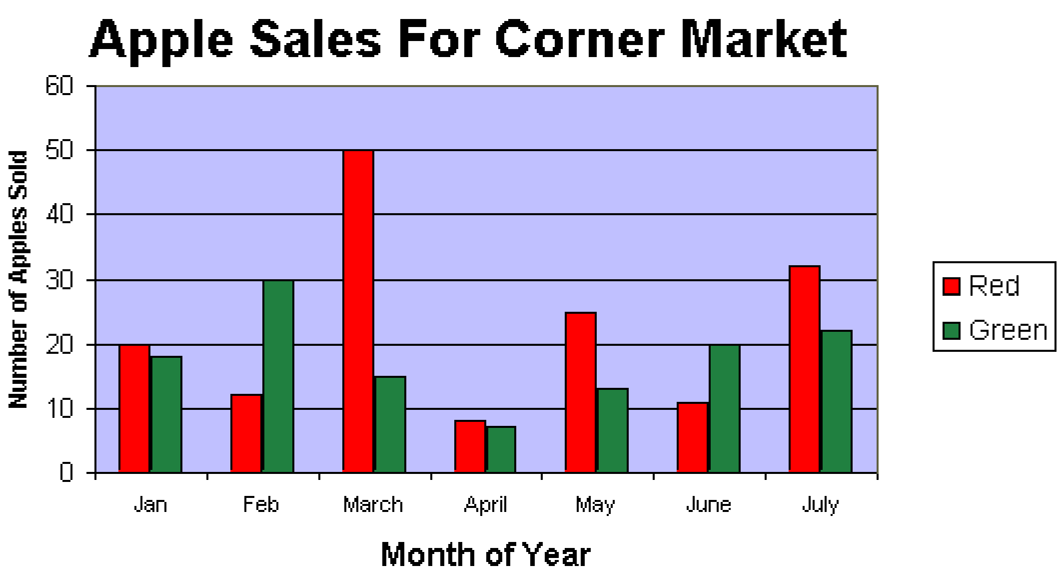
As you can see, the **area** of the larger rectangle is 9 times larger than the area of the smaller rectangle. So the area of the larger rectangle is 10 $feet^2$ × 9 or **90 $feet^2$**.

KCA #12 Interpret Graphs

KCA #12 Organizes, Interprets, Represents Data 7.4.2.k1
When graphing data, it is important to select the correct type of graph.

Bar Graphs

Variables that are distinct and unconnected between data points are represented by **bar graphs**. See the example below.



Question: How many more red apples were sold in January and March than green apples sold in February and June?

Red apples sold in January:	20	Green apples sold in February:	30
Red apples sold in March:	50	Green apples sold in June:	20
Total:	70	Total:	50

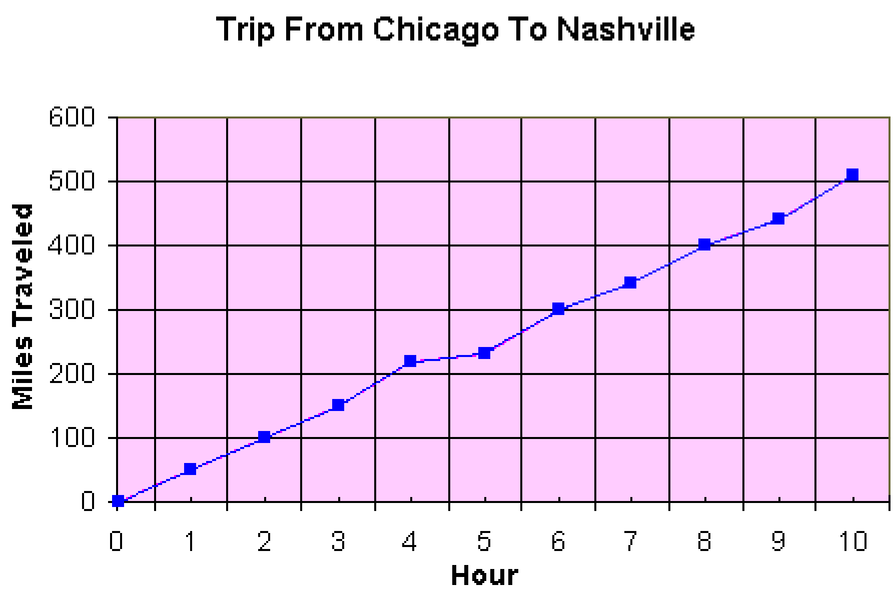
Subtract the totals to find how many more red apples were sold than green apples.

70 - 50 = 20

Answer: So, there were **20** more red apples sold in January and March than green apples sold in February and June.

Line Graphs

Variables that have continuous intervals that are unbroken sequences (e.g., growth of a plant) are represented by **line graphs**. See the example below.



Question: What was the average speed in miles per hour from hour 2 to hour 8, according to the graph?

At hour 2, the number of miles driven was 100.
At hour 8, the number of miles driven was 400.

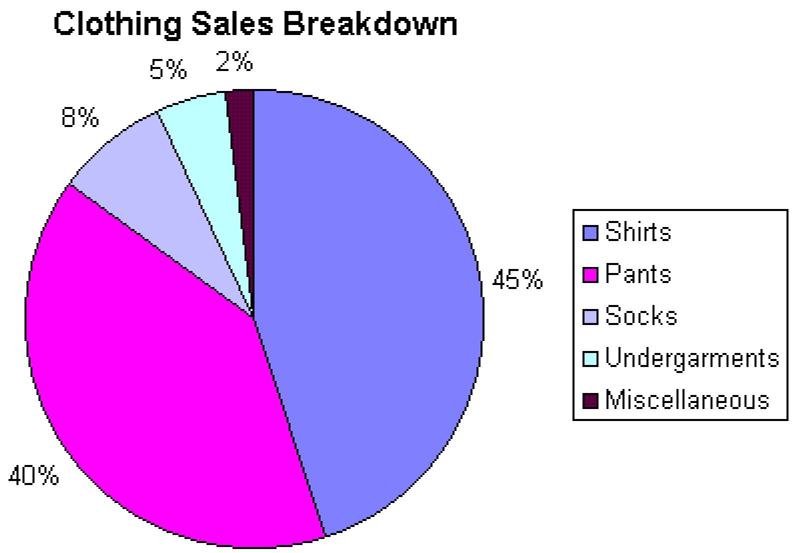
Calculate the miles per hour.

$$\frac{400 \text{ mi.} - 100 \text{ mi.}}{8 \text{ hr.} - 2 \text{ hr.}} = \frac{300 \text{ mi.}}{6 \text{ hr.}} = 50 \text{ mph}$$

Answer: So, the average speed from hour 2 to hour 8 was **50 miles per hour**.

Pie Graphs

A **pie chart** (or circle graph) is an excellent way to show how the relative sizes of the parts compare to each other and the whole. See the example below.



It is crucial to look at the legend when solving problems from a circle graph.

Question: The circle graph above shows the breakdown of today's sales at a department store. If the store made $1,000 today, how much of it was from selling pants?

Pant sales accounted for 40% of sales.

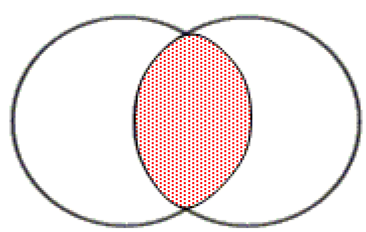
40% of $1,000 = 0.4 × $1,000 = $400

Answer: The store made **$400.00** from pants sales today.

Venn Diagrams

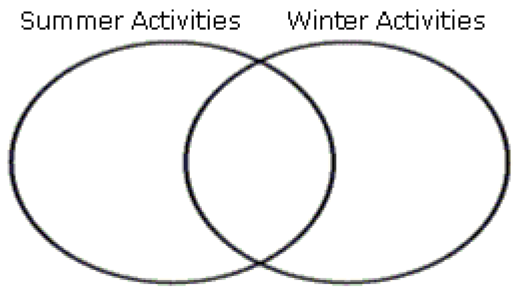
A **Venn diagram** is a way of showing how topics are the same and how they are different.

A Venn diagram is made up of overlapping circles. In this example, the shaded area is where the circles overlap.



Each circle is labeled on the outside with a topic.

The topics for this example are summer activities and winter activities.



Inside each circle, facts are listed for the topics. Facts are true statements about a topic. When a fact is listed in the overlapping area of the circles, this means the fact is true for both topics.

In this example, the facts “practice the piano” and “use my flashcard” belong in both summer and winter activities. All other facts belong under one topic or the other, but not both.

Box-and-Whisker Plots

A box-and-whisker plot is a method of displaying and interpreting a set of data. The data is first arranged in numerical order (that is, from lowest to highest) and then sorted into two equal parts at the median, or "middle point." If the data set has an odd number of entries, the median is the middle entry in the list. If the data set has an even number of entries, the median is equal to the sum of the two middle numbers divided by two.

Example 1:
The following set of data represents daily temperatures in a 7-day period:

85, 70, 75, 87, 83, 76, 80

The data set is arranged in numerical order from lowest to highest.

70, 75, 76, 80, 83, 85, 87

The median in this data set is 80 because this entry is exactly in the middle of the seven entries.

70, 75, 76, 80, 83, 85, 87
median is in red

Example 2:
The following set of data represents daily temperatures in a 10-day period:

60, 85, 70, 75, 90, 87, 83, 80, 65, 76

The data set is arranged in numerical order from lowest to highest.

60, 65, 70, 75, 76, 80, 83, 85, 87, 90

The median in the data set is 78, because the sum of the two middle numbers (76 and 80) divided by two equals 78.

60, 65, 70, 75, 76, (78), 80, 83, 85, 87, 90
median is in red

The data is further separated into quartiles, which divide the data into sections, each of which contains 25% of the total data.

The **first quartile** is located at the median (middle point) of the lower part of the data (that is, the numbers to the left of the median). Another name for the first quartile is the **lower quartile**. The data from the minimum value to the first quartile represents the first 25% of the data.

The **second quartile** is another name for the **median** of the entire set of data. The data from the first quartile to the median represents the second 25% of the data. The data from the median to the third quartile represents the third 25% of the data.

The **third quartile** is located at the median (middle point) of the upper part of the data (that is, the numbers to the right of the median). Another name for the third quartile is the **upper quartile**. The data from the third quartile to the maximum value represents the last 25% of the data.

The **interquartile range** is calculated by subtracting the 1st quartile from the 3rd quartile. It is the range of the middle 50% of a distribution.

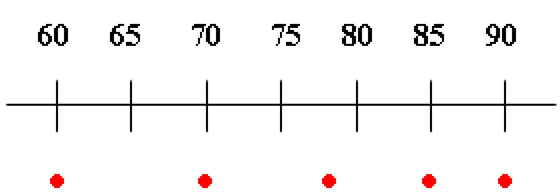
Example 3:

first quartile ↓ third quartile ↓

60, 65, 70, 75, 76, (78), 80, 83, 85, 87, 90

↑ median of all data

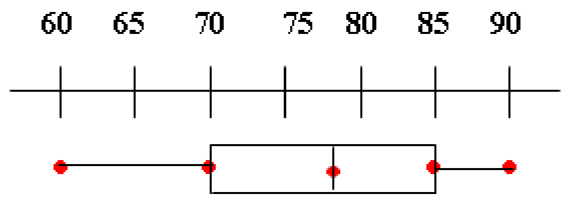
The data is placed on a number line. The first (lower) quartile, second quartile (median), and third (upper) quartile are noted by placing a point beneath each value. The smallest value (the minimum) and the largest value (the maximum) are also noted on the number line.



First quartile = 70
Second quartile = 78
Third quartile = 85
Smallest value = 60
Largest value = 90
Interquartile Range = 85 - 70 = 15

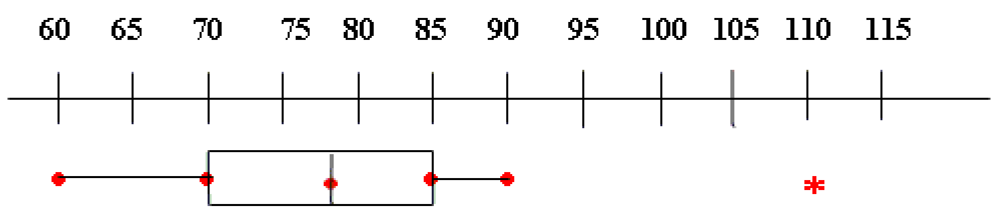
The box plot is completed by drawing a box with ends through the points of the first and third quartiles. A vertical line (top to bottom) is drawn through the box at the median (second quartile) point. Then, lines (called "whiskers") are drawn out from each end of the box to the smallest and largest values.

Example:



Sometimes one piece of data will be far outside the range of the other values. This single piece of data is called an “outlier”. It is marked with an asterisk (*) so it’s clear there are no data entries that fall in the range between the outlier and the smallest or largest value.

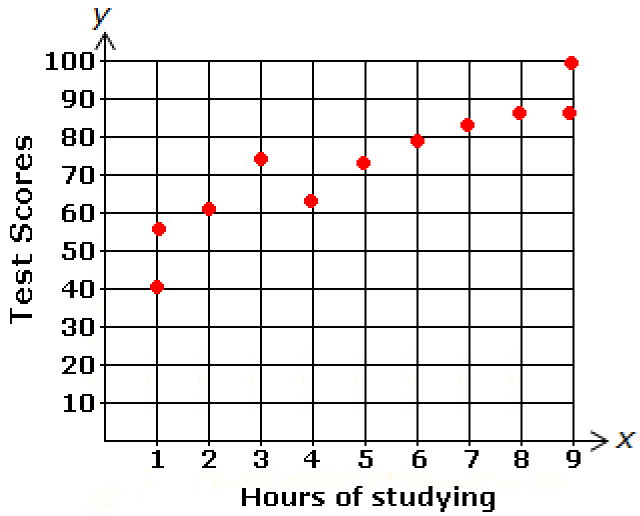
Example:



Scatter Plots

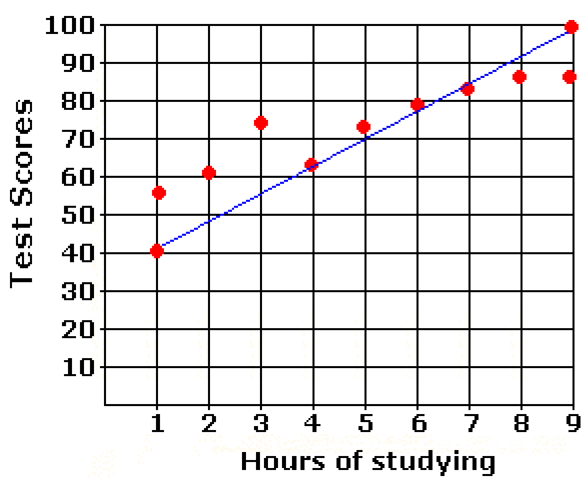
Scatter plots are used to examine two sets of data and to investigate the possible relationship (or correlation) between two variables. The pattern of the points suggests how closely the data is related.

Example: This scatter plot is used to determine the correlation between the number of hours a student studied and the scores on his tests.



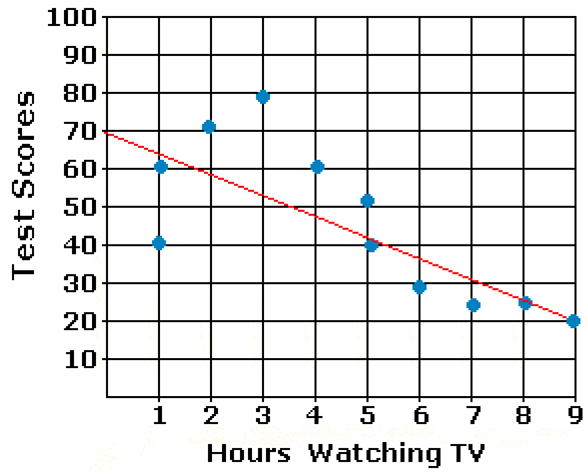
If the data displayed on the scatter plot forms a line that **RISES from left to right**, the variables are said to have a **positive correlation**. This "trend line" (or line of best fit) allows predictions to be made based on current data.

Example: The data on this scatter plot forms a line of best fit that RISES from left to right. The trend line in this scatter plot shows a positive correlation between the number of hours spent studying and scores on the tests.



If the data displayed forms a trend line that **FALLS from left to right**, the variables have a **negative correlation**.

Example: This data shows the more hours that were spent watching television, the lower the scores on the test, indicating a negative correlation between the number of television hours and test scores.



Stem-and-Leaf Plots

A stem-and-leaf plot is a method of organizing groups of data. Each number in the data is broken into a "stem" and a "leaf". The "leaf" is always the last digit in the number (the ones place) and the "stem" is the entire number except the last digit (the tens place; or the tens and hundreds place; or the tens, hundreds and thousands place).

Example:

Test Scores

Stem	Leaf
6	0 5 7
7	5
8	0 5 5 8 9
9	0 2 4 5 5 7
10	0 0 0

This stem-and-leaf plot shows test scores ranging from 60 through 100. The stems are the first digit (or digits) of the number, and the numbers that make up the leaf are the last digits. Stem 6 has three "leaf" numbers, which shows that the test scores are 60, 65 and 67.

6 | 0 = 60   6 | 5 = 65   6 | 7 = 67

Stem 10 also has three leaf numbers, each of which indicates the number is 100.

10 | 0 = 100

To create a stem-and-leaf plot, the number data first must be arranged from the lowest value to the highest value.

Example: Bob's scores on his last six math tests were 72, 58, 78, 65, 93 and 84. Arranged from lowest to highest, these scores are **5**8, **6**5, **7**2, **7**8, **8**4, **9**3.

The data numbers are placed on the plot as “stems” and “leaves”. Using the above scores, the stems will be the digits from 5 through 9, which are written vertically (up and down), with a line drawn to the right of the numbers.

Example:

5 |
6 |
7 |
8 |
9 |

The leaves are placed on the plot to the right of the line. Depending on the data, a stem may have more than one leaf.

Example:

Scores are 58, 65, 72, 78, 84, 93

The first value in the list is 58 so the stem is 5 and its leaf is 8.

5 | 8
6 |
7 |
8 |
9 |

The next value in the list is 65 so the stem is 6 and its leaf is 5.

5 | 8
6 | 5
7 |
8 |
9 |

Each leaf is placed next to its stem until all the data are entered. Note that stem 7 has two leaves.

5	8
6	5
7	2 8
8	4
9	3

KCA #13 Misleading Data 7.4.2.a3

KCA #13 Statistical Claims

Just because someone says something does not mean it is always true. Even if that someone works for a large corporation it doesn't mean the information is accurate and unbiased. Even if the results of the survey look very official and well-presented, there can be problems with the validity of the claims &##8211; "is it the truth?"

If information is **biased**, it means that your own personal opinion accounts for the way you handle information. If you love red cars, you could have a **bias for** red cars. If you hate black cars, you could have a **bias against** black cars.

Sometimes statistical data is presented in such a way as to give the impression of having a different outcome. Evaluating (thinking about) the validity of claims is important because it helps you look for the truth that might be buried within the information. It allows you to focus on the parts of the information that need more investigation because they might not be true.

Example: If a national survey says that 100% of people polled like black licorice ice cream (and you don't, and you don't know anyone who does either), it might be necessary to evaluate the validity of the claim by asking some questions. Who was surveyed? How many people did they survey? Did they toss out any parts of the results from people who did not like that flavor of ice cream? Did they only poll people standing in ice cream stores eating black licorice ice cream?

Identifying claims made in sampling and in statistics, and evaluating those claims, is an important part of analyzing data and working with statistics.

Representing Data

1. National statistics for movie viewing in small towns came out last week. They indicate that in towns with populations under 100,000, theaters averaged 1,000 tickets sold per week for the new children's movie, which is an average of 1% of the population. In Fairmont (population 95,000) the manager of the Bijou Theater told a news reporter that he had 3,150 children come see the new movie in one week. There is one showing per day of the movie at that theater. The Bijou has 150 seats. Why is the manager's claim not true?

The Bijou is a large theater.

The maximum number of seats that could be sold at the Bijou is 1,050.

The average number of tickets sold in Fairmont should be 950, or 1% of the population.

The national average of 1% is much too low.

* **Explanation**: With only one showing per day, and only 150 seats in the theater, 3,150 children could not have seen the movie over the week.

KCA #14 & #15 Number Patterns

KCA #14 and #15 Patterns & Nth Term 7.2.1.k1 & 7.2.1.k4
When there is a rule that describes the relationship between a set of different numbers, objects, or events, we call this a **pattern**. A **number pattern** is a rule that describes the relationship between a set of numbers. The rule can be used to predict the value of unknown numbers in the set.

Example 1

Look at the list of numbers below. What is the missing number?

65, 59, 53, 47, 41, __?__

To find the missing number, first find the pattern, or rule, that describes the relationship between the numbers.

The first number is 65. The next number is 59, which is **6 less** than 65. The next number is 53, which is **6 less** than 59. This pattern continues. So, we have found the rule: *Each number is 6 less than the previous number.*

So, the missing number is 6 less than 41: 41 - 6 = **35**.

Example 2

Look at the list of numbers below. What is the missing number?

$$3,\ 6,\ 12,\ \underline{\ ?\ },\ 48,\ 96$$

Find the rule: The first number is 3. The next number is 6, which is **3 more** than 3. The next number is 12, which is **6 more** than 6. Since the difference between each number changed, try to find a different pattern.

6 is **twice as much** as 3. 12 is **twice as much** as 6. So, we have found the rule: *Each number is double the previous number.*

So, the missing number is twice as much as 12: $12 \times 2 =$ **24**.

Example 3

Look at the list of numbers below. The 8th number in the list is missing. What is the missing number?

$$2,\ 11,\ 20,\ 29,\ 38,\ \_\_,\ \_\_,\ \underline{\ ?\ }$$

Find the rule: *Each number is 9 more than the previous number.*

So, the number that follows 38 is 47: $38 + 9 = 47$.
The number that follows 47 is 56: $47 + 9 = 56$.
The missing number, which follows 56, is 65: $56 + 9 =$ **65**.

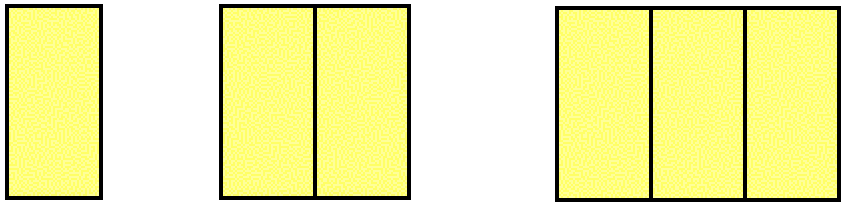
Geometric Patterns

When finding the missing picture in a list of pictures, figure out the pattern in the list.

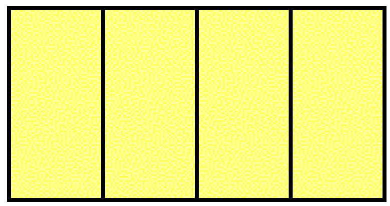
First, look at the change in the number of objects in each picture in the list. Then, figure out **how much bigger** or **how much smaller** each picture is than the picture before it.

Here are some common geometric patterns:

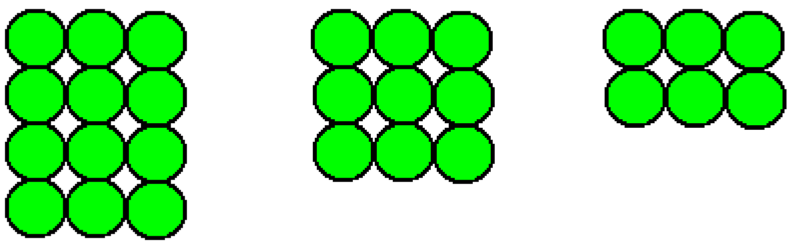
Sometimes the pictures **increase** by a certain amount. In this case, the number of rectangles increases by 1.



The next picture in the pattern would have 4 rectangles. It would look like this:



Sometimes the pictures **decrease** by a certain amount. In this case, the number of circles decreases by 3.



The next picture in the pattern would have 3 circles. It would look like this: